MINIMAL SURFACES IN HYPERBOLIC GEOMETRY

Winter School Côte d'Azur

Extra notes

Question: More "natural" conditions ensuring uniqueness of the minimal surface?

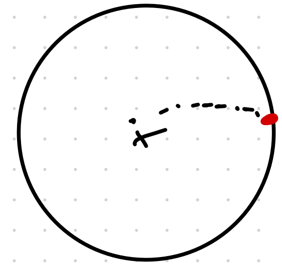
Let (M, h) complete hyperbolic mfd, $M \cong S \times \mathbb{R}$.

- (M, h) complete $\Longrightarrow (M, h) \cong \mathbb{H}^3 / \Gamma$

  $\Gamma < PSL_2(\mathbb{C})$ discrete, $\Gamma \cong \pi_1 S$

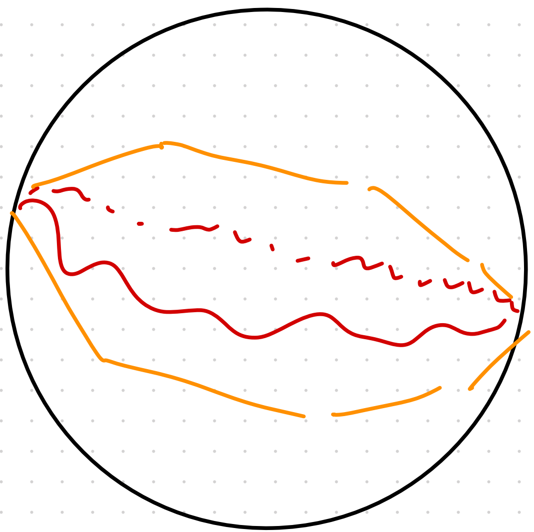
- lim't set

$$\Lambda(\Gamma) := \left\{ \begin{array}{c} \text{accumulation points} \\ \text{of } \Gamma \cdot x_0 \end{array} \right\} \subset \partial_\infty \mathbb{H}^3$$



- **Fact**: (M, h) is quasi-Fuchsian

  $\iff \Lambda(\Gamma)$ is a Jordan curve



"$\Leftarrow$" $\mathcal{C}(M)$: convex hull of $\Lambda(\Gamma) / \Gamma$

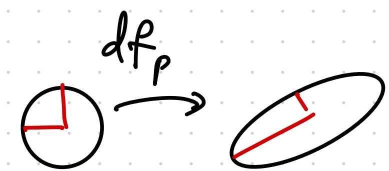
is geodesically convex and compact

- In fact, $\Lambda(\Gamma)$ is a quasi-circle

A map $f : \underset{\partial_\infty \mathbb{H}^3}{\overset{\shortparallel}{\mathbb{CP}^1}} \longrightarrow \underset{\partial_\infty \mathbb{H}^3}{\overset{\shortparallel}{\mathbb{CP}^1}}$ is K-quasiconformal

if f is absolutely continuous on lines
($\Rightarrow f$ differentiable almost everywhere)

and $$\operatorname*{ess\,sup}_{p \in \mathbb{CP}^1} \frac{\text{major axis of } df_p(S^1)}{\text{minor axis of } df_p(S^1)} < K$$



f 1-quasiconformal $\Longleftrightarrow$ f Möbius

Λ is a K-quasicircle $\Longleftrightarrow$ $\Lambda = f(\text{great circle})$
for f K-quasiconformal

<u>Thm</u> (S. '15)

There exists $C, \varepsilon_0 > 0$ such that for every $\Sigma \subset (M, h)$ with $\varepsilon \leq \varepsilon_0$,

minimal $\simeq S \times \{*\}$

$(1+\varepsilon)$ - qF

$$\sup_{p \in \Sigma} |\lambda(p)| \leq C \varepsilon$$

<u>Cor</u> If (M, h) is $(1 + \frac{1}{C})$-qF, then the minimal surface homotopic to $S \times \{*\}$ is <u>unique</u>.